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Bose–Einstein Condensation of Two-Dimensional Magnetoexcitons Under the Influence of Rashba Spin–Orbit Coupling

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This study is concerned with a two-dimensional electron–hole system in a strong perpendicular magnetic field with special attention devoted to the Rashba spin–orbit coupling. The influence of this interaction on the chemical potential of the Bose–Einstein condensed magnetoexcitons and on the ground state energy, and on the energy of the single-particle elementary excitations are investigated in the Hartree–Fock approximation. We demonstrate that chemical potential is monotonic function versus the value of the filling factor with negative compressibility, which leads to instability of the Bose–Einstein condensate of magnetoexcitons.

Keywords: Magnetoexciton, Bose–Einstein Condensation, Rashba Spin–Orbit Coupling.

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1. INTRODUCTION

The influence of the spin–orbit coupling (SOC) on the two-dimensional (2D) Wannier–Mott excitons in double quantum well (DQW) structures, as well as the possibilities of nonconventional electron–hole (e–h) pairing in these conditions were discussed in Refs. [1, 2]. The main results are breaking of the spin degeneracy of the electrons and holes, changes of the exciton structure, and new properties of the Bose–Einstein condensed excitons. There are two types of SOC. One of them described by Dresselhaus[3] is known to be intrinsically present in zinc blende structure. The Rashba spin–orbit coupling (RSOC)[4, 5] depends on the electric field strength E_z perpendicular to the layer surface. Side by side with questions related to the Bose–Einstein condensation (BEC) phenomenon, there exist a vast number of investigations in the field of spin–orbit coupling effects.[6–13] Since the mid 1980s, as was mentioned in Ref. [7], there has been extensive interest in the effects of an applied electric field normal to the layers on the optical properties of semiconductor quantum wells (QWs) and superlattices (SLs). The arising inversion asymmetry leads to anisotropic optical transitions.

The theoretical calculations of the Pockels effect for GaAs/$Ga_{1-x}Al_xAs$ and SLs demonstrate this statement.[7] In Ref. [6] it was underlined that the spin degeneracy of the electron and hole states is the combined effect of the inversion symmetry in space and time.

Nevertheless the Rashba spin splitting of 2D hole systems is very different from the more familiar case of 2D electron systems. In Ref. [6] this was explained by the fact that the holes have typically larger masses and smaller kinetic energies. The SOC is more important for holes than for electrons. When the carriers are moving through the inversion asymmetric potential, the spin degeneracy is removed even in the absence of an external magnetic field B. In this case there are two different branches of energy, $E_\uparrow(\vec{k}) \neq E_\downarrow(\vec{k})$, and the spin splitting is present. In quasi-2D QWs this spin splitting can be the consequence of a bulk inversion asymmetry (BIA) of the underlying crystal (for example, as in zinc blende crystals), or of a structure inversion asymmetry (SIA) of the confinement potential.

In both cases of inversion asymmetry the spin splitting takes place in the absence of B, i.e., $E_\uparrow(\vec{k}) \neq E_\downarrow(\vec{k})$, but the Kramers degeneracy continues to exist, $E_\uparrow(\vec{k}) = E_\downarrow(-\vec{k})$. This spin splitting is not due to the Zeeman effect because $B = 0$. In Ref. [6] the origin of the spin splitting is related with the motion of the electron through the inversion asymmetric spatial environment, the interaction with which is due to the SOC. The periodic parts of the electron Bloch functions are affected by the atomic fields that enter into the Pauli spin–orbit term, whereas the envelope functions are affected by the macroscopic environment. Following this picture, SIA leads to spin splitting, which is due to both the macroscopic electric

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field and the microscopic electric field from the atomic cores.

SIA spin splitting is always proportional to the macroscopic field strength times a prefactor depending on the microscopic spin–orbit interaction (SOI). This prefactor depends only on the matrix elements of the microscopic SOI and is due completely to the BIA. To reveal the origin of the spin splitting in a simpler way the following idea was suggested. One can imagine the electron moving with velocity $V_{||}$ to the plane of the layer subjected to the action of a perpendicular electric field E_z.

In the reference frame moving together with the electron the Lorentz transformation induces the magnetic field $B = (V_{||}/c)E_z$, which acts on the electron spin giving rise to such an indirect Zeeman effect. The estimations made showed that the spin splitting obtained in such a way is 5–6 orders of magnitude smaller than the experimentally observed values of the SOC. The discrepancy is due to the fact that the idea of Lorentz transformation neglects the contribution of the atomic cores to the SOI felt by Bloch electrons in a solid.[6] Another important detail, which must be remembered, is related to the crystallographic symmetry group of the solids. The spin splitting induced by the atomic cores, which is also called BIA splitting, also depends on the irreducible representations of the double group of the wavevector $\vec{k}$. For example, in the case of $\vec{k}$ parallel to the $\langle 111\rangle$ direction the wavevector group is C_{3v}. It has the double-group irreducible representations Γ_4, Γ_5 and Γ_6. In the case when the electron and light hole (LH) states transform according to the 2D representations Γ_4, whereas the heavy hole (HH) states transform according to the 1D representations Γ_5 and Γ_6, the BIA spin splitting vanishes for electrons and LHs and exists for HHs.[6] RSOC and intrinsic SOI under certain conditions lead to a Dirac cone formation out of a parabolic band and it is possible to create a "Mexican-hat-like" energy dispersion law.[8] The Mexican-hat-like dispersion has a line of degenerate low-energy points forming a ring. It can appear in a variety of physical systems. Such peculiarities were demonstrated in Refs. [1, 2]. The Mexican-hat-like dispersion law leads to a weak crystallization transition,[9] whereas in cold atom physics it gives rise to topologically different ground states of the Bose–Einstein condensed atoms and molecules.[8]

Now the Landau quantization of electrons and holes depending on their band structure will be discussed. In the calculations of Refs. [10, 11] of the hole Landau levels in the strained asymmetric p-type $GeSi_xGe_{1-x}$ QWs the cyclotron masses were determined. Self-consistent hole subband calculations were combined with calculations of the Landau levels using a $6\times 6\vec{k}\cdot\vec{p}$ Hamiltonian for the topmost Γ_8 and Γ_7 bulk valence bands.

To include the magnetic field in the calculation of the Landau levels the canonical momentum $\hat{p} = -i\hbar\vec{\nabla}$ is substituted by the kinetic momentum $\hat{\pi} = \hat{p} - q\vec{A}(\vec{r})/c$, where q is the electric charge of the quasi-particle.[6] These questions have been discussed in Ref. [14].

As was mentioned in Refs. [12, 13] the Rashba model can be described by purely group theoretical means. For an electron in an s-like conduction band the total angular momentum with SOI is $j = 1/2$. Both the wavevectors $\vec{k}$ and the electric strength $\vec{E}$ are polar vectors, whereas their cross product $[\vec{k}\times\vec{E}]$ is an axial vector. Its point product with the spin axial vector $\vec{\sigma}$ gives rise to the triple scalar product $[\vec{k}\times\vec{E}]\cdot\vec{\sigma}$. This expression is an invariant under the action of the group symmetry elements forming the identity representation Γ_1. Similar arguments were given in Ref. [15]. In the first quantization representation the wavevector $\vec{k}$ is substituted by $-i\vec{\nabla}$. In the Γ_6-type conduction band the triple scalar product is the only term of the first order in $\vec{\nabla}$ and $\vec{E}$ compatible with the symmetry of the band.

The electric field strength E_z depends on the density of charges in the syste.[12,13] The interaction constants $\alpha_e E_z$ and $\beta_h E_z$ were evaluated in Refs. [1, 2] for different values of E_z, arriving at the conclusion that at $E_z =$ 100–200 kV/cm the RSOC is a dominant mechanism for the energy band spin splitting. The main goals of the Refs. [1, 2] were to show that changing these parameters is an alternative method to examine the BEC of the 2DWannier–Mott excitons and their crossover from the low density regime with atom-like structure of the e–h pairs to the high density e–h pairs. In the latter case the electrons and holes form a Fermi degenerate gas. The weak interaction of the particles near the corresponding Fermi levels gives rise to their coherent pairing and to Bardeen–Cooper–Schrieffer (BCS)-type condensation.[16]

In the presence of the SOC the real excitonic order parameter is changed due to the mixing of the spin states. One of the most distinct effects, which is expected to take place in the frame of exciton condensation is the controllable mixing of the dark and bright exciton states.[1,2] Their mixing could lead to a change of the intensity of the coherent light emission. It was supposed that the ground exciton state is composed predominantly of the dark excitons, which do not couple to the light due to the total spin projections of the e–h pair being equal to ± 2. In the frame of this model the bright excitons with spins (± 1) are situated above the dark excitons. Similar questions will be discussed below in the case of 2D magnetoexcitons.

The aim of the present paper is to investigate properties of the electron–hole system beyond the Hartree–Fock–Bogoliubov approximation, taking into account the Rashba spin–orbit coupling. We consider three aspects of the problem: obtain the chemical potential of the BEC-ed magnetoexcitons, study the energy of the single-particle elementary excitations and the ground state energy.

2. HAMILTONIAN OF THE COULOMB ELECTRON–HOLE INTERACTION

The e–h Coulomb interaction we obtain below taking into account the influence of the RSOC in the frame of the

conduction and valence bands. The corresponding Bloch wavefunctions including their periodic parts are

$$\begin{aligned}|\psi_c(R_1,p;x,y)\rangle &= \frac{e^{ipx}}{\sqrt{L_x}}U_{c,s,p}(\vec{r})\begin{vmatrix} a_0\varphi_0(\eta_c) \\ b_1\varphi_1(\eta_c)\end{vmatrix}\\ |\psi_v(R_1,q;x,y)\rangle &= \frac{e^{iqx}}{\sqrt{L_x}}\frac{1}{\sqrt{2}}(U_{v,P,X,q}(\vec{r})\\ &\quad -iU_{v,P,Y,q}(\vec{r}))\begin{vmatrix} c_3\varphi_3(\eta_v) \\ d_0\varphi_0(\eta_v)\end{vmatrix}\\ \eta_c &= \frac{y}{l}-pl;\quad \eta_v=\frac{y}{l}-ql\end{aligned} \tag{1}$$

Here $U_{c,s,p}(\vec{r})$ is the s-type conduction band periodic part and $(1/\sqrt{2})(U_{v,P,X,q}(\vec{r})-iU_{v,P,Y,q}(\vec{r}))$ are the p-type valence band periodic parts. a_0, b_1, d_0, c_3 are the coefficients of spin–orbit interaction.[14]

The orthogonality between the conduction and valence electron Bloch wavefunctions is attained due to their orthogonal periodic parts, whereas the orthogonality of the wavefunctions belonging to the same bands and having the same periodic parts is reached due to different numbers of Landau quantization wavefunctions $\varphi_{c,n}(y,p)$ and $\varphi_{v,m}(y,p)$. The conduction and valence electrons have the same electric charge $-|e|$ and their dimensionless variables have the same structure $y/l-pl$ and $y/l-ql$. The last variable is $\frac{y}{l}+ql$ in the case of the hole wavefunction $\varphi_{h,n}(y,q)$ due to the positive value of the hole charge $|e|$.

The electron and hole states on the given Landau level are N manifold degenerated, where

$$N=\frac{S}{2\pi l^2};\quad l^2=\frac{\hbar c}{eH} \tag{2}$$

Here S is the surface area of the layer and $2\pi l^2$ is the area of the quantum cyclotron orbit with the radius $l\sqrt{2}$. Thus, N is the total number of the possible center of gyration positions on the surface of the layer.

The Hamiltonian of the Coulomb electron–hole interaction can be calculated in the same way as was demonstrated in the paper.[17] Differently from it we have dealt with spinor-type wavefunctions for electrons and holes with a column representation (1). The creation and annihilation operators for conduction and valence electrons are denoted as $a^\dagger_{c,R_i,p}, a_{c,R_i,p}, a^\dagger_{v,R_j,q}, a_{v,R_j,q}$. The Hamiltonian of their Coulomb interaction has the form:

$$\begin{aligned}\hat{H} &= -\mu_e\sum_p a^\dagger_p a_p-\mu_h\sum_p b^\dagger_p b_p\\ &\quad +\frac{1}{2}\sum_{p,q,s}F_{c-c}(c,R_i,p;c,R_i,q;c,R_i,p-s;c,R_i,q+s)\\ &\quad \times a^\dagger_{c,R_i,p}a^\dagger_{c,R_i,q}a_{v,R_i,q+s}a_{c,R_i,p-s}\\ &\quad +\frac{1}{2}\sum_{p,q,s}F_{v-v}(v,R_j,p;v,R_j,q;v,R_j,p-s;v,R_j,q+s)\\ &\quad \times a^\dagger_{v,R_j,p}a^\dagger_{v,R_j,q}a_{v,R_j,q+s}a_{v,R_j,p-s}\\ &\quad -\sum_{p,q,s}F_{c-v}(c,R_i,p;v,R_j,q;c,R_i,p-s;v,R_j,q+s)\\ &\quad \times a^\dagger_{c,R_i,p}a^\dagger_{v,R_j,q}a_{v,R_j,q+s}a_{c,R_i,p-s}\end{aligned} \tag{3}$$

Here the Coulomb matrix elements are determined as follows

$$\begin{aligned}&F_{c-v}(c,R_i,p;v,R_j,q;c,R_i,p-s;v,R_j,q+s)\\ &=\int d\vec{\rho}_1\int d\vec{\rho}_2\psi^{c*}_{R_i,p}(\vec{\rho}_1)\psi^{v*}_{R_j,q}(\vec{\rho}_2)\\ &\quad \times V_{12}\psi^{c}_{R_i,p-s}(\vec{\rho}_1)\psi^{v}_{R_j,q+s}(\vec{\rho}_2)\end{aligned} \tag{4}$$

where

$$V_{12}=\frac{e^2}{\varepsilon_0|\vec{\rho}_1-\vec{\rho}_2|} \tag{5}$$

Here the function $\psi^c_{R_i,p}$ are the envelope parts (1) of the Bloch functions. Their periodic parts being integrated on the elementary lattice cell, can be excluded from the final expressions of the desirable matrix elements, because the dipole moments of the band to band transitions are not needed now. The variables $\vec{\rho}_i$ are 2D vectors and ε_0 is the dielectric constant of the medium. The series expansion of the Coulomb interaction

$$V_{12}=\sum_{\vec{\kappa}}V_{\vec{\kappa}}\exp[i\vec{\kappa}(\vec{\rho}_1-\vec{\rho}_2)] \tag{6}$$

will be used, where $\vec{\kappa}=(\kappa_x,\kappa_y)$ is 2D wave vector and

$$V_{\vec{\kappa}}=V_{\kappa_x,\kappa_y}=\frac{2\pi e^2}{\varepsilon_0 S|\vec{\kappa}|}=\frac{2\pi e^2}{\varepsilon_0 S\sqrt{\kappa_x^2+\kappa_y^2}} \tag{7}$$

The matrix elements (3) have the following form:

$$\begin{aligned}&\sum_{p,q,s}F_{c-c}(c,R_i,p;c,R_i,q;c,R_i,p-s;c,R_i,q+s)\\ &=\sum_{\kappa_y}V_{s,\kappa_y}\exp\left[-\frac{(s^2+\kappa_y^2)l^2}{2}-i\kappa_y(q+s-p)l^2\right]\\ &\quad \times\left(|a_0|^2+(1-\frac{(s^2+\kappa_y^2)l^2}{2})|b_1|^2\right)^2\\ &\sum_{p,q,s}F_{v-v}(v,R_j,p;v,R_j,q;v,R_j,p-s;v,R_j,q+s)\\ &=\sum_{\kappa_y}V_{s,\kappa_y}\mathrm{Exp}\left[-\frac{(s^2+\kappa_y^2)l^2}{2}-i\kappa_y(q+s-p)l^2\right]\times\left(|d_0|^2\right.\\ &\quad \left.+\left(1-\frac{(s^2+\kappa_y^2)l^2(-12+(s^2+\kappa_y^2)l^2)(-6+(s^2+\kappa_y^2)l^2)}{48}\right)|c_3|^2\right)^2\\ &\sum_{p,q,s}F_{c-v}(c,R_i,p;v,R_j,q;c,R_i,p-s;v,R_j,q+s)\\ &=\sum_{\kappa_y}V_{s,\kappa_y}\mathrm{Exp}\left[-\frac{(s^2+\kappa_y^2)l^2}{2}+i\kappa_y(q+p)l^2\right]\end{aligned}$$

$$\times\left(|c_3|^2|a_0|^2\frac{48-(s^2+\kappa_y^2)l^2(-12+(s^2+\kappa_y^2)l^2)(-6+(s^2+\kappa_y^2)l^2)}{48}\right.$$
$$+|d_0|^2|b_1|^2\frac{(2-(s^2+\kappa_y^2)l^2)}{2}+|a_0|^2|d_0|^2+|c_3|^2|b_1|^2$$
$$\times(96-192(s^2+\kappa_y^2)l^2+108(s^2+\kappa_y^2)^2l^4$$
$$\left.-20(s^2+\kappa_y^2)^3l^6+(s^2+\kappa_y^2)^4l^8)\cdot(96)^{-1}\right) \quad (8)$$

3. GROUND STATE ENERGY, CHEMICAL POTENTIAL AND ENERGY OF THE SINGLE-PARTICLE ELEMENTARY EXCITATIONS

Consider the BEC of magnetoexcitons in a single particle state with wave vector $\vec{k}$ in the Hartree–Fock–Bogoliubov approximation (HFBA) and Rashba spin–orbit interaction. The energy of electrons and holes as well as their chemical potentials are measured relative to their lowest Landau levels. The exciton formation reaction $e+h \leftrightarrow ex$ implies the relation between the chemical potentials

$$\mu_e+\mu_h=\mu_{ex}=\mu \quad (9)$$

Below, we assume $\mu_e=\mu_h=\mu_{ex}/2$.

In the case of BEC of magnetoexcitons in Ref. [18] as a ground state wave functions was chosen the BCS-type wave function $|\psi_g(k)\rangle$ and as the excited wave functions the wave functions of the coherent excited states introduced in Ref. [18] for e–h systems in a similar way as it was done by Anderson[19] in the case of superconductors. The ground state wave function was introduced following Keldysh–Kozlov method[20] by the action of the displacement unitary transformation $\hat{D}(\sqrt{N_{ex}})$ on the vacuum state of the initially introduced electron–hole operators

$$|\psi_g(k)\rangle=\hat{D}(\sqrt{N_{ex}})|0\rangle; \quad a_p|0\rangle=b_p|0\rangle=0 \quad (10)$$

The coherent excited states were generated in Ref. [18] by the expression

$$|\psi^e(q\pm\frac{Q_x}{2})\rangle=a_{q+Q_x/2}a_{q-Q_x/2}|\psi_g(k)\rangle \quad (11)$$

The unitary transformation $\hat{D}(\sqrt{N_{ex}})$ breaks the gauge symmetry of the starting Hamiltonian (3) transforms it in a new Hamiltonian $\hat{D}H\hat{D}^\dagger$ gives to the ground state wave function $|\psi_g(k)\rangle$ and to macroscopic displacement $\sqrt{N_{ex}}$ of the exciton creation operator

$$d^\dagger(k)=\frac{1}{\sqrt{N}}\sum_t e^{-iQ_ytl^2}a^\dagger_{k_x/2+t}b^\dagger_{k_x/2-t} \quad (12)$$

In difference on the quantum optics and on Glauber Coherent states[21] the exciton creation and annihilation operators are not pure Bose operators but only quasi-boson operators.[22]

The unitary transformation

$$\hat{D}(\sqrt{N_{ex}})=\exp[\sqrt{N_{ex}}(d^\dagger(k)-d(k))] \quad (13)$$

leads to Bogoliubov u–v transformation

$$\alpha_p=Da_pD^\dagger=ua_p-v\left(p-\frac{k_x}{2}\right)b^\dagger_{k_x-p}$$
$$\beta_p=Db_pD^\dagger=ub_p+v\left(\frac{k_x}{2}-p\right)a^\dagger_{k_x-p} \quad (14)$$

as well as to inverse transform action

$$a_p=u\alpha_p+v\left(p-\frac{k_x}{2}\right)\beta^\dagger_{k_x-p}$$
$$b_p=u\beta_p-v\left(\frac{k_x}{2}-p\right)\alpha^\dagger_{k_x-p} \quad (15)$$

where

$$u=\cos g, \quad v=\sin g, \quad u^2+v^2=1$$
$$v(t)=ve^{-iK_ytl^2}; \quad g=\sqrt{2\pi l^2 n_{ex}}; \quad n_{ex}=\frac{N_{ex}}{S} \quad (16)$$

with the confinement of the lowest Landau levels (LLLs). It was shown that Ref. [18]

$$N_{ex}=v^2N; \quad n_{ex}=\frac{v^2}{2\pi l^2}; \quad g=v \quad (17)$$

where v^2 is the filling factor of the LLLs.

It leads to the relations $u=\cos v$ and $v=\sin v$, which can be satisfied only in the limit $v<1$. The theory developed in Ref. [18] and its application below can be made in the restriction $v<1$.

The Hamiltonian (3) after the unitary transformation (15) will contain operators $\alpha_p^\dagger, \alpha_p, \beta_p^\dagger, \beta_p$ in arbitrary ordering:

$$H=U+H_2+H' \quad (18)$$

The first term U does not contain operators α_p and β_p and plays the role of the new ground state energy. The second term H_2 is quadratic in the operators α_p and β_p and appears as a result of transpositions of the new operators and their normal ordering. In this transposition, the commutation relations of the Fermi operators α_p and β_p transform terms with four operators into quadratic terms. The term H' contains the remaining normal-ordered terms with four operators, which is treated as a perturbation. The term U can be represented as

$$U=-\mu N_{ex}-(1-v^2)N_{ex}I_{ex}(\vec{k})|a_0|^2|d_0|^2$$
$$-\frac{1}{2}v^2N_{ex}I_l|a_0|^4--\frac{1}{2}v^2N_{ex}I_l|d_0|^4$$
$$-\frac{3}{8}v^2N_{ex}I_l|b_1|^4-\frac{147}{512}v^2N_{ex}I_l|c_3|^4 \quad (19)$$

where $I_{ex}(\vec{k})$ is the exciton ionization potential:

$$I_{ex}(\vec{k})=I_le^{-k^2l^2/4}I_0\left(\frac{k^2l^2}{4}\right)$$

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Here I_l is the ionization potential of magnetoexciton within the lowest Landau levels approximation and equals to $e^2/\varepsilon l\sqrt{\pi/2}$, where l is magnetic length and ε is the background dielectric constant. $I_0(z)$ is the modified Bessel function.

The term H_2 contains diagonal quadratic terms as well as the terms describing the creation and annihilation of the new e–h pairs from the new vacuum state $|\psi_g(\vec{k})\rangle$. It has the form

$$H_2 = \sum_p E(\vec{k}, v^2, \mu)(\alpha_p^+\alpha_p + \beta_p^+\beta_p) - \sum_p \left[uv\left(\frac{k_x}{2} - p\right)\beta_{k_x-p}\alpha_p + uv\left(p - \frac{k_x}{2}\right)\alpha_p^+\beta_{k_x-p}^+ \right]\psi(\vec{k}, v^2, \mu) \tag{20}$$

where

$$E(\vec{k}, v^2, \mu) = -u^2v^2I_l|a_0|^4 - \frac{3}{4}u^2v^2I_l|b_1|^4 + v^4I_l|d_0|^4 + \frac{147}{256}I_lv^4|c_3|^4 + 2u^2v^2I_{ex}(\vec{k})|a_0|^2|d_0|^2 - \frac{\mu}{2}(u^2 - v^2) \tag{21}$$

and

$$\psi(\vec{k}, v^2, \mu) = -I_{ex}(\vec{k})(v^2 - u^2)|a_0|^2|d_0|^2 + \frac{3}{4}I_lv^2\left(\frac{49}{64}|c_3|^4 + |b_1|^4\right) + I_lv^2(|a_0|^4 + |d_0|^4) + \mu \tag{22}$$

It is seen from the Hamiltonian H_2 that the new quasi-particles described by the operators α_p, β_p can appear spontaneously from the new vacuum state as a pair with total momentum k_x, which coincides with the translational wave vector of the Bose–Einstein condensate of magnetoexcitons. Such terms in the Hamiltonian and the corresponding diagrams are called dangerous ones and make the new vacuum state unstable. To avoid this instability, the condition of compensation of the dangerous diagrams is used. In the Hartree–Fock–Bogoliubov approximation (HFBA), when only the dangerous diagrams in H_2 are taken into account, the condition of their compensation is

$$\psi(\vec{k}, v^2, \mu) = 0 \tag{23}$$

This condition determines the unknown parameter of the theory, namely the chemical potential μ of the system. In the HFBA it is

$$\mu = I_{ex}(\vec{k})(v^2 - u^2)|a_0|^2|d_0|^2 - \frac{3}{4}I_lv^2\left(\frac{49}{64}|c_3|^4 + |b_1|^4\right) - I_lv^2(|a_0|^4 + |d_0|^4) \tag{24}$$

With the help of μ we can determine self-consistently the ground state energy U and the energy of the single-particle elementary excitations, which in the given approximation are:

$$\begin{aligned} U &= v^2N_{ex}\left(\frac{1}{2}I_l(|a_0|^4 + |d_0|^4) - I_{ex}(\vec{k})|a_0|^2|d_0|^2\right) + \frac{3}{8}v^2N_{ex}I_l\left(\frac{49}{64}|c_3|^4 + |b_1|^4\right) \\ E(\vec{k}, v^2, \mu) &= \frac{1}{2}I_{ex}(\vec{k}) - \frac{1}{2}I_lv^2|a_0|^4 + \frac{1}{2}I_lv^2|d_0|^4 + \frac{3}{8}\left(\frac{49}{64}|c_3|^4 - |b_1|^4\right)I_lv^2 \end{aligned} \tag{25}$$

This result well agrees with the result of Ref. [18]. Indeed, if we assume that there is no spin–orbit interaction then by Ref. [14] $|a_0|^2 = |d_0|^2 = 1$ and $|c_3|^2 = |b_1|^2 = 0$, and we will get exactly the same expression for the chemical potential, the ground state energy U and the energy of the single-particle elementary excitations as in the Ref. [18].

One can remember that in the GaAs-type crystal the electron cyclotron energy $\hbar\omega_{ce}$ becomes comparable to and larger than the 2D Wannier–Mott exciton binding energy and at the same time the magnetic length l becomes smaller than the exciton Bohr radius just at the values $H \geq 7$ T.

Magnetoexcitons exist only in the range of high perpendicular magnetic field, therefore we will demonstrate our results with high magnetic and electric fields: $E_z = 10$ kV/cm and $H = 10$ T.

In Figure 1 is presented the chemical potential versus filling factor v^2 for different values of the wave vector $\vec{k}$. You can see that chemical potential is monotonic function with negative compressibility, which leads to instability of the Bose–Einstein condensate of magnetoexcitons. Similar result was obtained in the paper,[18] when was considered BEC of magnetoexcitons without taken into account Rashba spin–orbit coupling and excited Landau levels.

The energy of the single-particle elementary excitations and the ground state energy depends on the two parameter: one of them is the filling factor v^2, but another is the

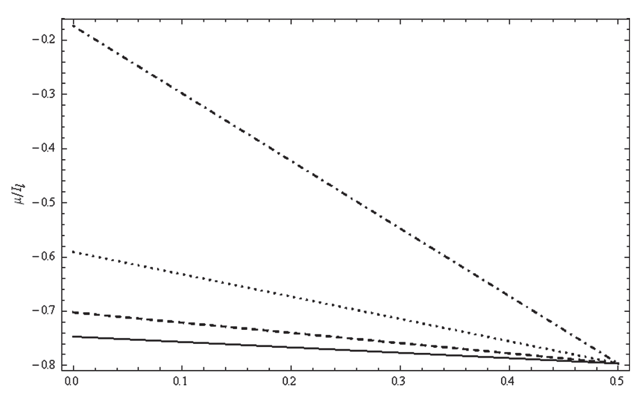

Fig. 1. Chemical potential versus filling factor v^2 for different values of the wave vector $\vec{k}$. Solid line: chemical potential of condensed excitons with $kl = 0$; Dashed line: the same, but for $kl = 0.5$; Dotted line: the same, but for $kl = 1$; Dot-dashed line: the same, but for $kl = 3.6$.

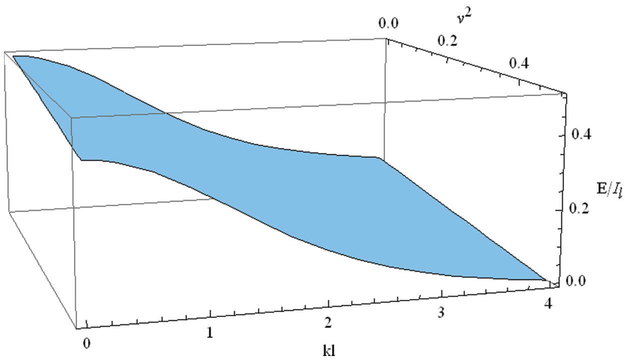

Fig. 2. The energy of the single-particle elementary excitations versus filling factor v^2 and wave vector $\vec{k}$.

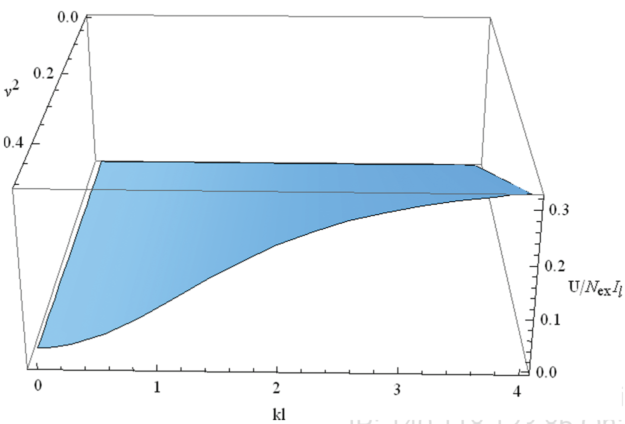

Fig. 3. The ground state energy versus filling factor v^2 and wave vector $\vec{k}$.

wave vector $\vec{k}$. Their behavior can see in Figures 2 and 3. With increasing values of the wavevector energy of the single-particle elementary excitations decreases asymptotically. The ground state energy has a reverse picture with increasing values of the wavevector ground state energy increasing with saturation. Also be noted that if the ground state energy increases with the increases filling factor, but the energy of the single-particle elementary excitations vice versa decreases.

4. CONCLUSION

We have studied the coherent pairing of electrons and holes in an ideal 2D structure in a strong transverse magnetic field under the influence Rashba spin–orbit coupling. The coherent pairing results in the Bose–Einstein condensation of 2D magnetoexcitons on the single-particle state with wave vector $\vec{k}$. The Keldysh–Kozlov–Kopaev method supplemented by the generalized random phase approximation was applied for the analysis. We have shown that chemical potential is monotonic function versus the value of the filling factor with negative compressibility, which leads to instability of the Bose–Einstein condensate of magnetoexcitons.

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